Il metodo assiomatico nelle scienze naturali

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The axiomatic method

Any scientific discipline has a (more or less precisely determined) domain of study, which it intends to investigate. The output of this investigation amounts to a partially organized collection of important properties and facts pertaining to the objects of this domain. Axiomatization should be the very effective methodology for *discovering*, *organizing*, *analysing*, *criticizing* and *communicating* the aforementioned body of theoretical and experimental data.

The axiomatic method has two kinds of purposes and benefits, which clearly overlap in many instances:

- the internal, *intradisciplinary*, ones, which are functional to the *development of the discipline* itself,

- the external, *interdisciplinary*, ones, which concern the *communication across different scientific communities*.

Intradisciplinary functions

An effective axiomatization of a discipline starts by identifying some *basic* (*primitive*) notions and by stating as axioms a *few, simple, clear* judgements concerning these. This, somewhat arbitrary, choice is motivated by the possibility of deducing as logical consequences many previously known facts, as well as new interesting facts. This allows for a deeper conceptual organization of the existing knowledge and a widening of the theory.

The requirement of producing a preliminary list of the basic primitive notions of a theory is a very reasonable request, albeit not so easy to achieve in general. On the other hand the refusal of producing it might hide *ambiguities* and *contradictions* and the unwillingness to face them explicitly.

The axiomatization process can help in detecting also ambiguities which one was unaware of in the original conception of the discipline. In fact it allows to achieve a rigorous account of what is assumed and is implied, preventing from smuggling in unconsciously *intentions* and *connotations* which are not explicitly mentioned.

We think that even what apparently may seem the most undesirable outcome of an axiomatization, i.e. *the derivation of a contradiction*, has in fact an invaluable importance for the understanding of the principles of the theory, and sometimes can have a revoutionary impact on the discipline itself. Such a contradiction is the evidence that some *misconception* was hidden in our intuition of the very basic notions of the discipline.

Cases in point are

- Frege's axiomatization of the notion of *class*,
- Linnæus' fixist theory of *species*.

  [this latter example is indirect, since Linnæus did not formulate an explicit axiomatic theory.]

Interdisciplinary functions

The fact that an axiomatization of a given field is based on *few, primitive, clearly isolated notions*, and *few, simple, concise statements*, helps in furthering the dialogue between scholars of diverse disciplines. Hence the axiomatic method is a medium for the fostering of constucitve criticism and it allows for the importation into the discipline of original suggestions stemming from even the most remote experiences. Moreover a rigorous axiomatic presentation reduces the possibility of those misunderstandings which often occur in interdisciplinary communication.

The important fact that, in principle, the axioms *contain already all* their logical consequences, can facilitate the difficult task of obtaining the conciseness necessary for a successful dialogue.

The axiomatization process provides the *most objective* formulation of a theory and this makes interested scholars less likely to be influenced by the authority of the authors or by their own prejudices upon them.

Sapiential aspects of axiomatization

Following Ennio De Giorgi, we maintain that a deep understanding of the foundational problems could unveil the *sapiential aspects* of the scientific activity. The conceptual clarity and the semiformal rigour of the traditional axiomatic method is incomparable in facilitating critical contributions of scholars from different fields. According to De Giorgi's ethical conception of scientific research, open and sincere dialogue among scholars of different attitudes, and "conviviality" in sharing of knowledge are the main factors of comprehension, friendship and mutual respect.

Although De Giorgi has been always ready to put himself into the struggle in defense of the fundamental human rights, nevertheless in his opinion theoretical analysis was much more relevant also in this respect. Illuminating of this attitude is his repeated affirmation that one of the most important axiomatic systems, one worth being studied in any order of schools, is the UDHR, the Universal Declaration of Human Rights of december 10th, 1948.

Limits of the axiomatic method

Any axiomatization gives only a *partial* account of the domain of investigation. This is rather clear for experimental Sciences, where only a small part of the interesting phenomena has a counterpart in the theory. But even in elementary Arithmetic we know, after Gödel, that

- *no effective axiomatization* can derive all truths involving sums and products of natural numbers.

Thus also in this case one has always to search for *new powerful* axioms, which can enhance the deductive strength of the theory.

One should always consider simultaneously *various different possible axiomatic frameworks* for any single discipline, whose various aspects and facts are often difficult to organize in a unified framework, based on few previously isolated primitive notions. A critical analysis and comparison of these systems may lead to a new synthesis, possibly based on new primitives, accomodating simultaneously most of the relevant data under consideration.

De Giorgi's Main Principles

- *Non-reductionism*

- *Open-endedness*

- *Semiformal rigor*

can be epitomized by the words of Hamlet: *There are more things in heaven and earth, . . . , than are dreamt of in your philosophy*.

Non-reductionism

There are many kinds of *qualitatively* different objects and concepts which are studied in the Sciences and Humanities. The mathematical modelling of many scientific concepts has been fruitful in providing a *quantitative analysis* of these concepts. But a total reduction of *natural scientific notions* to their mathematical codings can undermine the conceptual clarity of the notions themselves. Instead, the possibility of using *qualitatively*, and not only *quantitatively* different kinds of objects simplifies the enterprise of introducing new notions, especially in the Sciences of Nature.

For instance, *biological objects* are better defined *per se* and not on the basis of a decomposition into simpler constituents, whose mere sum might not measure up to an adequate definition of the biological object itself.

An abstract concept, like that of *species* cannot be adequately accounted for as a *set of individuals*, since it has properties, and it partakes in relations, as a *single complex* entity, independently of the specific set of individuals present at any given time. A scientific study of a species, in effect, has to consider its interactions with the *whole* ecosystem, and also its historical development.

Even in pure Mathematics, where the twentieth century's *set-theoretic reductionism* has obtained very important results, a "totalitary" interpretation may render it difficult, even impossible at times, to formulate appropriate axioms and conjectures.

- The intuitive notion of *operation* subsumes the *intensional* concept of computation process, and so operations cannot be simply coded by their graphs, or reduced to input-output diagrams.
- Taking *natural numbers* as primitives allows for a clearer analysis of the connections between their different implementations, not only as Von Neumann ordinals, but also as Frege-Russell cardinals, Church numerals, etc.
- Conceiving *collections* as *truth-valued operations* would force unnecessary commitments on the definition of collection, and yet it would not make apparent their intrinsic *extensionality*.

General non-reductionist axiomatic theories can appear *prima facie* unnecessarily complicated, but this drawback seems unescapable, if we want to represent *naturally* within the framework, at least in principle, the multitude of existing conceptual systems.

Open-endedness

An axiomatic framework should be open to extensions in any conceivable direction. The natural introduction of any sufficiently analyzed and clarified concept should be always possible. In axiomatizing a specific discipline, primitive notions and axioms should be formulated so as to allow for the introduction of *new objects* and *concepts* which can emerge from theoretical and experimental developments of the discipline. And this goal should possibly be achieved in a *conservative* way, without having to revise the formulation of the main properties previously axiomatized.

In particular, when axiomatizing a general foundational theory, one should look for a framework suitable for accommodating most of classical and modern theories arising in Mathematics, Logic, Computer Science, as well as in Physics, Biology, Economics, Linguistics, etc.

And the axiomatization of any particular science has to take into account the whole range of its theoretical and experimental achievements. As a matter of fact, it is often difficult, even impossible at times, to integrate the huge quantity of new information within the current conceptual framework of the theory.

Semiformal rigor

Foundations, like any scientific discipline, can be expounded in a rigorous, yet informal style, and investigated using the axiomatic method of traditional Mathematics. This seems of particular importance in most applied sciences, because these have not yet developed formal languages of their own.

In order to allow for critical analysis by scholars from different fields of research, a *rigorous* presentation using *natural language*, appears to be preferable to a *formal* axiomatization, carried out within some *artificial* language from Mathematical Logic. Of course, one has to render the statements as unambiguous as possible, by giving a precise axiomatic determination of the basic technical terms.

Also in Mathematics, where formal languages are well developed and formal axiomatizations in First Order Logic have become customary, it turns out that *none* of them can be satisfactorily taken as definitive. In fact, any formalization misses part of the intended meaning of the original theory.

The above considerations are not intended to sound as a *depreciation* of the relevance formal theories have in the development of Sciences. We term our approach *semi*formal rather than *in*formal, precisely because he thougut that any interesting axiomatization should admit *easy, natural* translations in suitable formal languages, thus benefiting from various important results of modern Mathematical Logic.

More importantly, the *formalizations* of the axiomatizations of different disciplines can exhibit *formal* (structural) similarities and relations, and fruitfully suggest *substantial* analogies and connections between the original notions, not apparent before.

De Giorgi's foundational programme

De Giorgi's foundational programme was developed around his weekly seminar at the Scuola Normale Superiore in Pisa. Foundations were not intended to give *safe and unquestionable grounds* to scientific theories, but rather to provide *conceptual environments* where research and criticism, in all sciences, could be carried out rigorously, but without the artificial constraints deriving from exasperate reductionism and formalization.

The premature death of Ennio De Giorgi in 1996 slowed down the development of the programme, but we hope that it can be maintained alive.

The axiomatic systems of 2000

During the last years of De Giorgi's life, the scope of the foundational programme widened, and centred on the formulation of general axiomatic frameworks suitable for expressing (*engrafting*) Mathematics and Logic, Informatics, Biology, Economics and, possibly, any sufficiently clear conceptual domain arising in the Sciences and Humanities. To this aim, only a few fundamental notions are isolated:

- *qualities, relations, operations,* and *collections.*

The primitive notions

Most disciplines in the Sciences and Humanities deal, ultimately, with *several qualitatively different* objects and study *properties, relations* and *operations* over them. Therefore, in order to design such a general axiomatic framework, it seems appropriate to select the *pre-mathematical* notions of *quality* (or *property)* and *relation* assuming only them as *primitive*, i.e. not reducible to preceding concepts.

Most axiomatizations of scientific theories make use of two other primitive kinds of objects: *operations* and *collections* (and sets). According to the basic pattern for engrafting new concepts in the general axiomatic framework, we introduce these notions by means of suitable *qualities*, which classify the objects under consideration, and of suitable *relations*, which describe their behaviour and connect them with the objects introduced previously. Thereafter, also operations and collections will be used together with the other basic notions for axiomatizing new theories.

A serious problem arises in dealing with *experimental sciences*, and especially Biology, within a foundational framework *à la* De Giorgi: dew to its origin in Mathematics, Logic and Computer Science, this framework accommodates naturally only abstract, "eternally invariant" objects. A few attemps have been made of engrafting "variation", at the prize of some (perhaps cumbersome) parametrizations.

We refer only to the introduction of the concept of *variable* of classical Mathematical Physics suggested in [Var94], and to the treatment of *contingency* and *modalities* outlined in [ont99].

However much more is needed for superseding the obstacles met in axiomatizing biological notions, in particular when dealing with *living objects*: for instance, a lion is a living object until it dies, it is in the relation $Rcomp$ with its stomach until it is removed, etc.

To be sure, the very primitive notion of biological object seems to bring about a kind of dependence on *time* and *space* parameters, thus leading to a general notion of *variable object*.

The *variable objects*

We start from the naive idea that the objects of natural sciences (say the $Earth$, or the famous policedog Rex) have an *individual* nature wich goes beyond the *singular concrete things* with which they are identified at each instant (usually a suitable finite aggregate of particles). Of course, depending on the objects under consideration, not only *time* instants or intervals come into play, but also *spatial* determinations and more generally any kind of relevant data suitable for characterizing a given situation.

Therefore we introduce a very large and unspecified collection $\mathcal{S}$ of parameters or "sorts" (the *states* of the universe), conceived to encompass all "phase spaces" considered by any scientific theory under consideration, in particular here Biology. At suitable values of the parameters, any natural object has a "concrete representative" (the *Earth* is an aggregate of determined molecules, and *Rex* of determined cells). Depending on the state, these concrete representatives *change* (some atoms of the Earth are eliminated or substituted, some old cells disappear in generating new ones inside Rex, etc.), while the "abstact object" is still considered to remain the *same*.

We introduce

Qvar, the quality of being a *variable* object

$\mathcal{S}$, the collection of the *sorts* or *states*

Ract, the relation of *actualization*

AXIOM 1 *$Qvar$ is a quality, $\mathcal{S}$ is a collection, and $Ract$ is a ternary relation s.t.*

$Ract\ x, y, z \implies Qvar\ x \ \wedge\ y \in \mathcal{S}$;

$Ract\ x, s, y \ \wedge\ Ract\ x, s, z \implies y = z$.

If X is a variable object, $s \in \mathcal{S}$ is a sort, and $Ract\ X, s, z$, then z is denoted by X^s and called the *actualization of X at the state s*. If X has an actualization X^s at the state s we say that X *exists at* s, and that X^s *is X at* s.

One has to investigate the connections between a variable object and its instantiations, and the structure of the "phase space" $\mathcal{S}$ (as a matter of fact, this topic is still largely unexplored).

We consider only one example, typical in Biology: the case of an operation acting on a variable object and producing another variable object. It seems convenient to assume that any such (abstract) operation can be "decomposed" into several (concrete) operations which, at suitable states, act on suitable (instantiations of) objects, and produce other (instantiations of) objects, possibly in new states. The first object has to be (a component of) the argument and the last one (a component of) the result of the abstract operation.

Operations on variable objects

AXIOM 2 *If F is an operation and X, Y are variable objects such that $FX = Y$, then there exist*

- *operations $f_1, \dots, f_n$,*
- *variable objects $V^1, \dots, V^n$, and*
- *states $s_1, \dots, s_n \in \mathcal{S}$ such that*

$$f_i V^i_{s_i} = V^{i+1}_{s_i} \quad for \quad i = 1, \dots, n-1.$$

Moreover $\mathrm{Rcomp}\ X, V^1$ and $\mathrm{Rcomp}\ Y, V^n$.

The operations $f_1, \dots, f_n$, are called the *components* of the variable operation F.

We have considered only simple operations for sake of brevity, but clearly we intend to postulate a correspondent axiom for binary operations as well. Moreover binary operations can actually appear as components of simple variable operations and *vice versa*. (Alternatively, we could consider binary operations as acting on *pairs* of objects.)

Similar axioms should be given also for relations, qualities, or collections involving variables objects. Finally it should be appropriate to specify suitable relations among the states involved in such axioms.

Self-description

A crucial instance for a foundational framework is the possibility of *self-description*:

- the most relevant operations and relations which the framework utilizes should themselves be first class objects in the framework.

For instance, various *qualities, relations* and *operations* should be introduced in order to classify and describe the behaviour of the different *kinds* of objects, including qualities, relations, operations themselves.

A FRAMEWORK THEORY

The *Primitives*

- *the object* q *is a quality*;
- *the object* r *is a binary relation*;
- *the object* s *is a ternary relation*;
- *the object* t *is a quaternary relation*;
- *the object* x *has the quality* q (written as $q\,x$);
- *the objects* x,y *are in the binary relation* r (written as $r\,x,y$);
- *the objects* x,y,z *are in the ternary relation* s (written as $s\,x,y,z$);
- *the objects* x,y,z,w *are in the quaternary relation* t (written as $t\,x,y,z,w$).

The *classifying qualities*

In accordance to the general principle of *selfdescription* we introduce and axiomatize four distinguished qualities corresponding to the primitive kinds of objects:

- *Qqual*, the quality of being a *quality*
- *Qrelb*, the quality of being a *binary relation*
- *Qrelt*, the quality of being a *ternary relation*
- *Qrelq*, the quality of being a *quaternary relation*

The *fundamental relations*

We introduce, next, the *fundamental relations* which describe the behaviour of qualities and relations:

- *Rqual*, the relation giving the *behaviour of qualities*
- *Rrelb*, the relation giving the *behaviour of binary relations*
- *Rrelt*, the relation giving the *behaviour of ternary relations*

The *axioms*

AXIOM 3

Qqual, Qrelb, Qrelt, Qrelq are qualities. No object has simultaneously two of them.

AXIOM 4

x is a quality $\Longleftrightarrow$ *Qqual x;*

x is a binary relation $\Longleftrightarrow$ *Qrelb x;*

x is a ternary relation $\Longleftrightarrow$ *Qrelt x;*

x is a quaternary relation $\Longleftrightarrow$ *Qrelq x.*

AXIOM 5 $Rqual$ *is a binary relation s.t.* $Rqual\ q, x \Longleftrightarrow Qqual\ q \wedge q\, x.$

AXIOM 6 $Rrelb$ *is a ternary relation s.t.* $Rrelb\ r, x, y \Longleftrightarrow Rrelb\ r \wedge r\ x, y.$

AXIOM 7 $Rrelt$ *is a quaternary relation s.t.* $Rrelt\ s, x, y, z \Longleftrightarrow Rrelt\ s \wedge s\ x, y, z.$

The *identity relation*

Qualities and relations have an essentially *intentional* character, hence we do *not* postulate *extensionality* for them. More generally, since we have to deal with objects of *any* imaginable kind, it appears inappropriate to give explicit *criteria* of equality. Hence we introduce axiomatically an *identity* relation *R_id* between objects of arbitrary kind:

AXIOM 8 Rid *is a binary relation s.t.*
$Rid\ x, y \Longleftrightarrow x$ *and* y *are the same object.*

Notation: $x = y$ stands for $Rid\ x, y$.

CAVEAT Apparently the action of *quaternary relations* is left here without internal description: it would require a *quinary* relation R_rel_q, thus giving rise to an infinite sequence of fundamental relations of increasing arities, as in [**?**]. Stopping at a low level of complexity, an internal description of quaternary relations can equally well be obtained in various ways, following [**?**, **?**] or [**?**, **?**].

The *operations*

The concept of *operation* builds solely on the intuition of an operation as an object which *acts* (operates) on one, or two, objects and possibly produces a *result*. Hence we introduce the qualities $Qops$ and $Qopb$ of being respectively a *simple* (unary) and a *binary operation* and the corresponding relations $Rops$ and $Ropb$. The *functionality* of operations is expressed by postulating that the relations $Rops$ and $Ropb$ are "univalent".

AXIOM 9 *$Qops$, $Qopb$ are qualities, $Rops$ is a ternary relation, $Ropb$ is a quaternary relation.*
1. If $Rops\ x, y, z$ then $Qops\ x$;
2. If $Rops\ f, x, y$ and $Rops\ f, x, z$, then $y = z$.
3. If $Ropb\ x, y, z, w$ then $Qopb\ x$;
4. If $Ropb\ g, x, y, z$ and $Ropb\ g, x, y, w$ then $z = w$.

Notation:
$fx = y$, $fxy = w$ stand for $Rops\ f, x, y$ and $Ropb\ f, x, y, w$

Caveat: No extensionality for operations!

There may exist operations, acting on the same objects and giving the same result on each object, which are nonetheless different, for they operate according to different procedures.

The *collections*

The concept of *collection* aims to capture the most general notion of *"aggregation into a whole of different objects of any kind"*, which underlies the definitions of *class* and *set*, given by Frege and Cantor. They are *extensional* in nature, in the sense that collections having the same members are identical.

- *Qcoll*, the quality of being a *collection*
- *Rcoll*, the *membership* relation
- *Rincl*, the *inclusion* relation

AXIOM 10 *$Qcoll$ is a quality and $Rcoll$ is a binary relation s.t.* $Rcoll\ x, y \implies Qcoll\ x$

AXIOM 11 *$Rincl$ is a binary relation. If C, D are collections, then:*

1. $Rincl\ C, D \iff \forall x\, (Rcoll\ D, x \implies Rcoll\ C, x)$*;*

2. $Rincl\ C, D \wedge Rincl\ D, C \implies C = D$*.*

Notation:

$x \in C$ stands for $Rcoll\ C, x$

$C \supseteq D$ and $D \subseteq C$ stand for $Rincl\ C, D$

We postulate the existence of four fundamental collections, namely the *universal* collection V, the *empty* collection $\emptyset$, the collection of *all* collections $Coll$, and the collection of all *sets* Ins.Sets are isolated as *"small collections which can be freely manipulated"*, differently from general collections, which are *"too large and complicate"*. They comprehend all collections commonly considered in the applied Sciences, and in general all (intuitively) *finite* collections.

AXIOM 12 *$V, \emptyset, Coll, Ins$ are collections:*
1. for all objects x, $x \in V$ and $x \notin \emptyset$;
2. $x \in Coll$ if and only if $Qcoll\ x$;
3. Ins is a subcollection of $Coll$, whose elements are all sets.

A thorough development of the theory of operations and collections is carried out in [**?**, **?**]. We do not introduce other operations or collections here, since our aim is only that of presenting a *minimal* axiomatic framework, suitable for axiomatizinging many different disciplines.

Engraftings

Flexibility and capacity of these axiomatic systems is proved by the axiomatizations of various basic concepts stemming from diverse disciplines. Significant examples are the engraftings of:

- *variables* from classical Mathematical Physics in [Var94],
- *collections, sets* and *functions* in [FHL99],
- *partial operations* as general computational processes in [FHL95],
- *propositions*, *predicates*, and various notions of *truth* in [CP95, con-, FL97],
- *living objects, species* and other notions of Biology in [GF99, FFG].

Il processo di assiomatizzazione appare soprattutto un valido mezzo per approfondire ed unificare diversi rami del sapere umano, fornendo al contempo un linguaggio abbastanza preciso e flessibile per un fruttuoso dialogo con studiosi di altre discipline; ci pare quindi appropriato concludere rileggendo i paragrafi finali di [con-]:

È difficile programmare l'esplorazione del mondo vario, ricco, colorato che dovrebbe ampliare il "paradiso di Cantor". Si può solo dire che il successo dell'esplorazione dipenderà probabilmente dal numero e dalla varietà degli "esploratori", dalla loro sensibilità nell'apprezzare le migliori tradizioni e le maggiori conquiste culturali del passato unita alle capacità di intuire quali potrebbero essere le innovazioni piú valide e piú feconde. Soprattutto sembra necessario unire fantasia e rigore scientifico, intendendo il rigore nel significato piú ampio che è stato delineato in [2000], ove si afferma:

Ogni ricerca sugli assiomi fondamentali di Matematica, Logica e Informatica, come pure delle diverse scienze sperimentali, umanistiche, filosofiche comporta fra l'altro il superamento di una visione troppo chiusa delle diverse specializzazioni ed un'idea piú ampia del rigore matematico o scientifico. Il rigore matematico non è solo accuratezza nelle dimostrazioni ma anche impegno a esporre nel modo piú chiaro e comprensibile i problemi che si vorrebbero risolvere, i teoremi che si vorrebbero dimostrare, le congetture che si vorrebbero verificare o confutare. Noi riteniamo che il rigore scientifico consista soprattutto nell'esporre chiaramente e liberamente le proprie certezze e i propri dubbi, i problemi che si ritiene di aver risolto e quelli che si vorrebbe risolvere o vedere risolti, evitando solo quei discorsi confusi, oscuri, inutilmente complicati che finiscono con l'annoiare anche l'ascoltatore meglio disposto. In ultima analisi dobbiamo concludere che ogni discorso sul metodo scientifico, sul rigore scientifico e sul significato della Scienza ci riporta alla fine alle pi antiche intuizioni sui valori sapienziali dell'umiltà, della "convivialità" (che insieme condivisione del sapere, amicizia, ricerca di reciproca comprensione) e della fiducia nella Sapienza che viene incontro a coloro che la amano e la cercano.

Programming the exploration of the various, rich, colourful world that should enlarge "Cantor's paradise" is a difficult task. One can only say that the success of the exploration will probably depend on the number and variety of the "explorers", on their ability of appreciating the best traditions and the greatest cultural achievements of the past, in conjunction with the faculty of imagining which innovations might be most fruitful and valuable. It seems utmost necessary to combine imagination and scientific rigor, intending rigor in the broader sense outlined in [2000], where we assert:

> Any investigation of the fundamental axioms of mathematics, logic and computer science, as well as of various experimental, human and philosophical sciences needs, among other things, to overcome a too restricted vision of the different specialities, and requires a broader idea of mathematical and scientific rigor. Mathematical rigor is not only carefulness of the proofs, but also engagement in exposing in the most clear and understandable way the problems one would want to solve, the theorems one would want to prove, the conjectures one would verify or refute. We think that scientific rigor mainly consists in clearly and frankly exposing their own certainties and doubts, which problems one believes to have solved and which one would like to solve or see solved, while avoiding those confuse, obscure, uselessly complicated talks that end up in annoying even the most favourably disposed listener.
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> Summing up, we may conclude that any consideration on method, rigor, and meaning of science leads us in the end to the ancient intuitions of the sapiential value of humility, of "conviviality" (which keeps together sharing of knowledge, friendship, search of mutual understanding), and of trust in Wisdom, which meets all those who love and seek it.

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