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The Foundational Programme of Ennio De Giorgi

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De Giorgi's foundational programme was developed mainly by a small group of researchers during his weekly seminar at the Scuola Normale Superiore in Pisa. From the very beginning, Foundations were not intended to give *safe and unquestionable grounds* to mathematical theories, but rather to provide *conceptual environments* where research and teaching, in mathematical sciences, could be carried out rigorously, but without the artificial constraints deriving from exasperate reductionism and formalization.

The premature death of Ennio De Giorgi in October '96, slowed down the development of the programme, which however is still fully alive. The state of the affairs at the time of his death is faithfully represented in [18].

De Giorgi's Main Priciples

can be epitomized by the words of Hamlet:
There are more things in heaven and earth,
. . . , *than are dreamt of in your philosophy*.

- *Non-reductionism*

- *Open-endedness*

- *Self-description*

- *Semiformal approach*

Non-reductionism

There are many kinds of *qualitatively* different objects and concepts which are studied in the Sciences and Humanities. Although the mathematical modelling of many scientific concepts has been fruitful in providing a quantitative analysis of these concepts, nevertheless reducing *natural scientific notions* to their mathematical codings can undermine the conceptual clarity of the notions themselves. Even in pure Mathematics, the twentieth century *set-theoretic reductionism*, whereby *natural numbers* are identified to finite *Von Neumann ordinals*, *ordered pairs* to *Kuratowski doubletons*, *relations* and *operations* to *graphs*, makes it difficult, even impossible at times, to formulate appropriate axioms and conjectures. On the other hand, for instance, taking *natural numbers* as primitives allows for a clearer analysis of the connections between their different implementations as Frege-Russell cardinals, Von Neumann ordinals, Church numerals, etc. Similarly, the intuitive notion of *operation* subsumes the *intensional* concept of computation process, and so operations cannot be simply coded by their graphs. On the other hand, conceiving *collections* as *truth-valued operations* forces unnecessary commitments on the definition of collection, and yet it does not make apparent their intrinsic *extensionality*.

Instead, the possibility of using *qualitatively*, and not only *quantitatively* different kinds of objects simplifies the enterprise of introducing new notions, especially in the Sciences of Nature. General non-reductionist axiomatic theories can appear *prima facie* unnecessarily complicated, but this drawback seems unescapable, if we want to represent *naturally* within the framework, at least in principle, the multitude of existing conceptual systems.

For instance in [23], *biological objects* are defined *per se* and not on the basis of a decomposition into simpler constituents, whose mere sum might not measure up to an adequate definition of the biological object itself. An abstract concept, like that of *species* cannot be adequately accounted for as a *set of individuals*, since it has properties, and it partakes in relations, as a *single complex* entity, independently of the specific set of individuals present at any given time. A scientific study of a species, in effect, has to consider its interactions with the *whole* ecosystem, and also its historical development.

Open-endedness

An axiomatic framework should be open to extensions in any conceivable direction. The natural introduction of any sufficiently analyzed and clarified concept should be always possible. In axiomatizing a specific discipline, primitive notions and axioms should be formulated so as to allow for the introduction of *new objects* and *concepts* which can emerge from theoretical and experimental developments of the discipline. And this goal should possibly be achieved in a *conservative* way, without having to revise the formulation of the main properties previously axiomatized.

In particular, when axiomatizing a general foundational theory, one should look for a framework suitable for accommodating most of classical and modern theories arising in Mathematics, Logic, Computer Science, and possibly Physics, Biology, Economics, Linguistics, etc.

As a matter of fact, it is often difficult, even impossible at times, to integrate the huge quantity of new information within the current conceptual framework of the theory.

Self-description

A crucial instance for a foundational framework is the possibility of *self-description*: the most relevant operations and relations which the framework utilizes should themselves be first class objects in the framework. For instance, various *qualities, relations* and *operations* should be introduced in order to classify and describe the behaviour ofthe different *species* of objects, including qualities, relations, operations themselves. Also the *assertions* and the *predicates* which arise in describing the framework should be objects in the framework itself.

Semiformal approach

Foundations, like any scientific discipline, should be expounded in a rigorous, yet informal style, and investigated using the axiomatic method of traditional Mathematics. In order to allow for critical analysis by scholars from different fields of research, a *rigorous* presentation using *natural language*, appears to be preferable to a *formal* axiomatization, carried out within some *artificial* language from Mathematical Logic. Of course, one has to render the statements as unambiguous as possible, by giving a precise axiomatic determination of the basic technical terms.

This seems of particular importance in many applied sciences, because these have not yet developed formal languages of their own. Also in Mathematics, where formal languages are well developed and formal axiomatizations in First Order Logic have become customary, it turns out that *none* of them can be satisfactorily taken as definitive. In fact, any formalization misses part of the intended meaning of the original theory.

The above considerations are not intended to sound as a *depreciation* of the relevance formal theories have in the development of Sciences. De Giorgi termed his approach *semi*formal rather than *in*formal, precisely because he thougut that any interesting axiomatization should admit *easy, natural* translations in suitable formal languages, thus benefiting from various important results of modern Mathematical Logic. More importantly, the *formalizations* of the axiomatizations of different disciplines can exhibit *formal* (structural) similarities and relations, and fruitfully suggest *substantial* analogies and connections between the original notions, not apparent before.

The axiomatic method

Any scientific discipline has a (more or less precisely determined) domain of study, which it intends to investigate. The output of this investigation amounts to a partially organized collection of important properties and facts pertaining to the objects of this domain. Axiomatization should the very effective methodology for *discovering, organizing, analysing, criticizing* and *communicating* the aforementioned body of theoretical and experimental data.

The axiomatic method has two kinds of purposes and benefits, which clearly overlap in many instances:

- the internal, *intradisciplinary*, ones, which are functional to the *development of the discipline* itself,

and

- the external, *interdisciplinary*, ones, which concern the *communication across different scientific communities*.

Intradisciplinary functions

An effective axiomatization of a discipline starts by identifying some *basic* (*primitive*) notions and by stating as axioms a *few, simple, clear* judgements concerning these. This, somewhat arbitrary, choice is motivated by the possibility of deducing as logical consequences many previously known facts, as well as new interesting facts. This allows for a deeper conceptual organization of the existing knowledge and a widening of the theory.

The requirement of producing a preliminary list of the basic primitive notions of a theory is a very reasonable request, albeit not so easy to achieve in general. On the other hand the refusal of producing it might hide *ambiguities* and *contradictions* and the unwillingness to face them explicitly.

The axiomatization process can help in detecting also ambiguities which one was unaware of in the original conception of the discipline. In fact it allows to achieve a rigorous account of what is assumed and is implied, preventing from smuggling in unconsciously *intentions* and *connotations* which are not explicitly mentioned.

De Giorgi thought that even what apparently may seem the most undesirable outcome of an axiomatization, i.e. the derivation of a contradiction, has in fact an invaluable importance for the understanding of the principles of the theory, and sometimes can have a revoutionary impact on the discipline itself. Such a contradiction is the evidence that some *misconception* was hidden in our intuition of the very basic notions of the discipline.

Cases in point are Frege's axiomatization of the notion of *class*, and Linnæus' fixist theory of *species*. Clearly this latter example is indirect, since Linnæus did not formulate an explicit axiomatic theory.

Interdisciplinary functions

The fact that an axiomatization of a given field is based on few, primitive, clearly isolated notions, and few, simple, concise statements, helps in furthering the dialogue between scholars of diverse disciplines. Hence the axiomatic method is a medium for the fostering of constucitve criticism and it allows for the importation into the discipline of original suggestions stemming from even the most remote experiences.

The fact that, in principle, the axioms contain already all their logical consequences, allows for obtaining the conciseness necessary for a successful dialogue. Moreover a rigorous axiomatic presentation reduces the possibility of those misunderstandings which often occur in interdisciplinary communication.

The axiomatization process provides a *more objective* formulation of the theory and this makes interested scholars less likely to be influenced by prejudices on the authors or their authority.

Sapiential aspects of axiomatization

De Giorgi maintained that a deep understanding of the foundational problems could unveil the *sapiential aspects* of the sientific activity. The conceptual clarity and the semiformal rigour of the traditional axiomatic method is incomparable in facilitating critical contributions of scholars from different fields.

Therefore the foundational programme fitted perfectly with De Giorgi's ethical conception of scientific research. Open and sincere dialogue among scholars of different attitudes, the conviviality in sharing of knowledge, are the main factors of comprehension, friendship and mutual respect.

Although De Giorgi has been always ready to put himself into the struggle in defense of the fundamental human rights, nevertheless in his opinion theoretical analysis was much more relevant also in this respect. Illuminating of this attitude is his repeated affirmation that one of the most important axiomatic systems, one worth being studied in any order of schools, is the UDHR, the Universal Declaration of Human Rights of 10/12/1948.

Historical development of De Giorgi's programme

1978-1984: Set-theoretic approach

1984-1989: From the *Teoria Quadro* to the *Teoria Ampia*

1990-1995: The *Basic Theories* and the *Teoria '95*

1996: The axiomatic systems of 2000

The Set-theoretic Approach

Old-style theories, hence (see [2])

- with *Urelemente*, so as to avoid set-theoretic reductionism;
- with *Large classes* which may be elements of sets, so as to provide a satisfying self-description;

- without the Axiom of Foundation, so as to leave open space to self-membership and other useful non-well-foundednesses.

The Free Construction Principle (see [1])

In order to satisfy his need of flexibility and naturality within an essentially set-theoretic framework, De Giorgi was led to formulate his *Principle of Free Construction*

FCP *It is always possible to define a set of sets by freely assigning its elements through an arbitrary parametrization.*

$$\forall f.\exists g.\forall x \in \text{dom } f.\ g(x) = \{g(y) \mid y \in f(x) \cap \text{dom } f\} \cup (f(x) \setminus \text{dom } f)$$

The Teoria Quadro [3]

- Five kinds of fundamental objects: *Classes*, *Pairs*, *Natural numbers*, *Uruples*, and *Uroperations*.

- Very strong self-descriptive power obtained by “playing the differenr keyboards” .

- A long list of supplementary axioms which, taken alltogether are inconsistent, leaving the task of finding maximal consistent sublists (the “problem of selfreference” of [4]).

The Teoria Ampia [5]

- More than 250 fundamental objects and more than 350 axioms.
- A complete treatment of the main mathematical entities together with the logical means of description.

In particular

- The (qmr)-structures, i.e. pairs (q,r) where q is a quality (classifying a kind of objects) and r is a relation describing the behaviour of this kind of objects.
- relational pairs (r,x) where r is a relation and x is an object of suitable kind. They code the extensions, namely those objects y s.t. yrx.
- The fundamental (q,r)-structure $(qrp, rginc)$ of relational pairs with generalized inclusion.

Unexpected Difficulties (see [6, 7])

The work of G. Lenzi and others on the relative consistency of the Ample Theory led to a surprising conclusion:

- The theory itself was relatively consistent, but it was pure chance (or rather exceptional farseeing of its inventor)!

  In fact

- almost all the natural extensions proposed by the authors were inconsistent, and moreover

- also seemingly innocent purely combinatorial relations could not be consistently adjoined.

Openendedness was badly violated!

The Basic theories

As a reaction to the unhappy exits of the Teoria Ampia, De Giorgi started to isolate very small fragments, namely

- The theories "$7+5$", "7×2", "5×7" and others where only few objects are explicitly named and no means of general constructions of new objects are given (see [8]).

This process ended with the *Teorie Base* of [10], where only those *basic* objects are introduced which should be subsequently used for engrafting the different theories, and only the *basic* axioms are postulated which are needed for ensuring their general behaviour. Since most disciplines in the Sciences and Humanities deal, ultimately, with *several qualitatively different* objects and study *properties, relations* and *operations* over them, It seemed appropriate to select the following fundamental notions:

- *qualities, relations, operations, collections* and *natural numbers.*

Engraftings

Flexibility and capacity of these axiomatic systems is proved by the axiomatizations of various basic concepts stemming from diverse disciplines. Significant examples are the engraftings of:

- *variables* from classical Mathematical Physics in [11],
- *collections, sets* and *functions* in [21],
- *partial operations* as general computational processes in [20],
- *propositions*, *predicates*, and various notions of *truth* in [15, 18],
- *living objects, species* and other notions of Biology in [23, 24].

The axiomatic systems of 2000

During the last years of De Giorgi's life, the scope of his foundational programme widened, and centred on the formulation of general axiomatic frameworks suitable for expressing (*engrafting*) not only Mathematics and Logic but also Informatics, Biology, Economics and, possibly, any sufficiently clear conceptual domain arising in the Sciences and Humanities.

Clear symptoms of this attitude are:

- Natural numbers lose their central role;

- Propositions and predicates become the main descriptive tools;

- The notion of truth takes the center of the stage.

The primitive notions

In order to design such a general axiomatic framework, the *pre-mathematical* notions of *quality* (or *property)* and of *binary, ternary, quaternary relation* are isolated, and only the following notions are assumed as *primitive*, i.e. not reducible to preceding concepts:

- *the object* q *is a quality*;
- *the object* r *is a binary relation*;
- *the object* s *is a ternary relation*;
- *the object* t *is a quaternary relation*;
- *the object* x *has the quality* q (written as $q\,x$);
- *the objects* x,y *are in the binary relation* r (written as $r\,x,y$);
- *the objects* x,y,z *are in the ternary relation* s (written as $s\,x,y,z$);
- *the objects* x,y,z,w *are in the quaternary relation* t (written as $t\,x,y,z,w$).

The *classifying qualities*:

In accordance to the general principle of *selfdescription* we introduce and axiomatize four distinguished qualities and relations corresponding to the primitive kinds of objects considered above

- *Qqual*, the quality of being a *quality*
- *Qrelb*, the quality of being a *binary relation*
- *Qrelt*, the quality of being a *ternary relation*
- *Qrelq*, the quality of being a *quaternary relation*

The *fundamental relations*:

We introduce, next, the *fundamental relations* which describe the behaviour of qualities and relations

- *Rqual*, the relation giving the *behaviour of qualities*
- *Rrelb*, the relation giving the *behaviour of binary relations*
- *Rrelt*, the relation giving the *behaviour of ternary relations*

AXIOM 1

Qqual, Qrelb, Qrelt, Qrelq are qualities. No object has simultaneously two of them.

AXIOM 2

x is a quality $\iff$ *Qqual x;*

x is a binary relation $\iff$ *Qrelb x;*

x is a ternary relation $\iff$ *Qrelt x;*

x is a quaternary relation $\iff$ *Qrelq x.*

AXIOM 3 $Rqual$ *is a binary relation s.t.* $Rqual\ q, x \iff Qqual\ q \wedge q\, x.$

AXIOM 4 $Rrelb$ *is a ternary relation s.t.* $Rrelb\ r, x, y \iff Rrelb\ r \wedge r\ x, y.$

AXIOM 5 $Rrelt$ *is a quaternary relation s.t.* $Rrelt\ s, x, y, z \iff Rrelt\ s \wedge s\ x, y, z.$

Apparently the action of *quaternary relations* is left here without internal description: it would require a *quinary* relation R_rel_q, thus giving rise to an infinite sequence of fundamental relations of increasing arities, as in [10]. Stopping at a low level of complexity, an internal description of quaternary relations can equally well be obtained in various ways, following [14, 15] or [18, 21].

Qualities and relations have an essentially *intentional* character, hence we do *not* postulate *extensionality* for them. More generally, since we have to deal with objects of *any* imaginable kind, it appears inappropriate to give explicit *criteria* of equality. Hence we introduce axiomatically an *identity* relation *R_id* between objects of arbitrary kind:

AXIOM 6 Rid *is a binary relation s.t.* $Rid\ x, y \Longleftrightarrow x$ *and* y *are the same object.*

Notation: $x = y$ stands for $Rid\ x, y$.

Most axiomatizations of scientific theories make use of two other primitive kinds of objects: *operations* and *collections* (and sets). According to the basic pattern for engrafting new concepts in the general axiomatic framework, we introduce suitable *qualities*, which classify the objects under consideration, and suitable *relations*, which describe their behaviour and connect them with the objects introduced previously. Operations and collections will be used together with the other basic notions for axiomatizing new theories.

Operations

The concept of *operation* builds solely on the intuition of an operation as an object which *acts* (operates) on one, or two, objects and possibly produces a *result*. As in the case of relations, we do not exclude that there are operations which act on more than two objects at once; we simply don't introduce them here for the sake of brevity. Hence we introduce the qualities $Qops$ and $Qopb$ of being respectively a *simple* (unary) and a *binary operation* and the corresponding relations $Rops$ and $Ropb$. The *functionality* of operations is expressed by postulating that the relations $Rops$ and $Ropb$ are "univalent".

AXIOM 7 *$Qops$ is a quality, $Rops$ is a ternary relation.*

1. If $Rops\ x, y, z$ then $Qops\ x$;

2. If $Rops\ f, x, y$ and $Rops\ f, x, z$, then $y = z$.

AXIOM 8 *$Qopb$ is a quality, $Ropb$ is a quaternary relation.*

1. If $Ropb\ x, y, z, w$ then $Qopb\ x$;

2. If $Ropb\ g, x, y, z$ and $Ropb\ g, x, y, w$ then $z = w$.

Notation:

$fx = y$ stands for $Rops\ f, x, y$

$fxy = w$ stands for $Ropb\ f, x, y, w$

Caveat: No extensionality for operations!

Collections

The concept of *collection* aims to capture the most general notion of “aggregation into a whole of different objects of any kind”, which underlies the definitions of *class* and *set*, given by Frege and Cantor. They are *extensional* in nature, in the sense that collections having the same members are identical. On the contrary, we have *not* postulated extensionality for operations, since we do not want to rule out the possibility that there exist operations, acting on the same objects and giving the same result on each object, which are nonetheless different, for they operate according to different procedures.

- *Qcoll*, the quality of being a *collection*
- *Rcoll*, the *membership* relation
- *Rincl*, the *inclusion* relation

AXIOM 9 *$Qcoll$ is a quality and $Rcoll$ is a binary relation s.t.*

$$Rcoll\ x, y \implies Qcoll\ x$$

AXIOM 10 *$Rincl$ is a binary relation. If C, D are collections, then:*

1. $Rincl\ C, D \iff$

$$\forall x\, (Rcoll\ D, x \implies Rcoll\ C, x);$$

2. $Rincl\ C, D\ \wedge\ Rincl\ D, C \implies C = D.$

Notation:

$x \in C$ stands for $Rcoll\ C, x$

$C \supseteq D$ and $D \subseteq C$ stand for $Rincl\ C, D$

We postulate the existence of four fundamental collections, namely the *universal* collection V, the *empty* collection $\emptyset$, the collection of *all* collections Coll, and the collection of all *sets* Ins.Sets are isolated as "small collections which can be freely manipulated", differently from general collections, which are "too large and complicate". They comprehend all collections commonly considered in the applied Sciences, and in general all (intuitively) *finite* collections.

AXIOM 11 *$V, \emptyset, Coll, Ins$ are collections:*

1. for all objects x, $x \in V$ and $x \notin \emptyset$;

2. $x \in Coll$ if and only if $Qcoll\ x$;

3. Ins is a subcollection of $Coll$, whose elements are all sets.

A thorough development of the theory of operations and collections is carried out in [18, 21]. We do not introduce other operations or collections here, since our aim is only that of presenting a *minimal* axiomatic framework, suitable for axiomatizinging many different disciplines.